Large Scale Electromagnetic Scattering Problems Solved Using The Locally Corrected Nyström Method and Adaptive Cross Approximation

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Outline

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Motivation

- Solutions of the large-scale electromagnetic scattering problems are typically obtained with Rao-Wilton-Glisson (RWG) Method of Moments (MoM) accelerated with Multi-Level-Fast-Multipole-Algorithm (MLFMA).

- There are challenges
  1. Inefficient error control of solution beyond 2-3 digits
  2. Poor conditioning of the matrix resulting from multiscale discretization of the model and/or the presence of highly resonant phenomena
  3. The necessity to repeat solution for each excitation

- The first challenge is addressed by geometry representation based on nonuniform rational b-spline (NURBS) surfaces and high-order locally corrected Nystrom (LCN) discretization

- The second and third challenges are addressed through fast direct solution of the matrix equation resultant from LCN discretization of the CFIE based on the block-LU decomposition aided with Adaptive-Cross-Approximation (ACA)
Motivation: Presence of High Attenuation

- Consider the analytic Mie-series solution for a perfect electric conducting (PEC) material
- The solution which is current $J$, has dynamic range of about 10,000
- Therefore, for a method to provide 2 digits of precision at the low-end part of the solution, it has to be more accurate at the high-end solution

Dipole at 5GHz

$2m = 33.33\lambda$

$0.25m$

Time snapshot at $t=0$ sec
Motivation: Presence of High Attenuation

RWG MoM Accelerated with Multilevel Fast Multipole Algorithm (MLFMA)

2.77 Million RWG Unknowns
9 Triangles per Wavelength

3.63 Million RWG Unknowns
11 Triangles per Wavelength

Average Relative Error over 20°

https://www.cemworks.com
Remedy to High Attenuation: High-Order (HO) Techniques

- High-Order Locally Corrected Nystrom (LCN) method is used as the discretization method\(^1\)
- Sharp edges are completely eliminated by using geometry modeling using non-uniform rational b-spline (NURBS) surfaces due to \(G^2\) continuity
- 92,928 Order 10 Unknowns (\(h_{\text{max}}=3.4\lambda\)) 384 Elements

\[ n = a \cdot \left( \frac{P^2}{c(\frac{P}{P_{\text{ref}}})} \right)^{\frac{P}{P_{\text{ref}}}} \cdot \frac{P^2}{P^{(\frac{3P}{P_{\text{ref}}})}}. \]

- Number of unknowns can be formulated analytically according to order and the desired accuracy
- We can predict the optimal number of unknowns according to the desired accuracy

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Fast Matrix Fill

- Fast direct solver based on IERUS’s V-Lox CEM software
- Adaptive Cross Approximation (ACA) for matrix fill and LU decomposition
- Multi-threaded and Multi-GPU accelerated with Out-of-Core option
- Mesh clustering required to achieve matrix block structure for fast compressed block-LU decomposition

- Clustering accomplished using binary tree method
- Cluster trees formed for test groups and source groups
- Self/Near blocks are not compressed
- Far blocks are compressed based on admissibility criterion, exploiting low rank property of blocks

\[ Z_{ts} = U_{ts} V_{ts}^H \quad \text{where} \quad Z_{ts} \in \mathbb{C}^{m \times n} \]

is rank \( k < \min (m, n) \)
Fast Direct Solve

- LU decomposition compressed with modified ACA based on compressed Z
- Standard LU formulas with compressed block representations substituted as appropriate

\[
L_{ts} = \left( Z_{ts} - \sum_{k=1}^{s-1} L_{tk} U_{ks} \right) U_{ss}^{-1}
\]

\[
U_{ts} = Z_{ts} - \sum_{k=1}^{t-1} L_{tk} U_{ks}
\]

Compressed Forms

\[
Z_{ts} = U_{ts} V_{ts}^H
\]

\[
L_{tk} = U_{L_{tk}} V_{L_{tk}}^H
\]

\[
U_{ks} = U_{U_{ks}} V_{U_{ks}}^H
\]

• ACA requires row/column generation and is done with efficient order of operations to reduce FLOP count
GPU Acceleration

- GPUs provide massive parallelism for significant application speed-up
- LU decomposition is accelerated via multiple GPU threads
- LU blocks are assigned to GPU threads
- To maximize throughput, full block is generated on GPU

- Block fed into ACA algorithm for compression
- Full blocks computed similarly
- Block throughput is optimized to maximize GPU utilization even for Out-of-Core operation
- Extremely complex, real-world geometry with >1.5M RWG unknowns solved in 1.1 days

![Graph showing speed-up with increasing number of GPUs]

500k RWG unknown problem on up to 8x Tesla K10 compared to 32-thread CPU
Study of Accuracy: Exact Sphere at 100MHz

Geometry: Analytical Cube-to-Sphere Mapping

Solution: Surface Current Density

Mean Relative Error vs. Max Relative Error
Current Produced by MFIE
Reference Solution: Mie-Series

CPU Acceleration vs. GPU Acceleration
Current Produced by CFIE (α=0.5)
Reference Solution: Mie-Series
Study of Accuracy: Torus

100MHz (outer radius=0.73\(\lambda\))

Geometry: NURBS Generated Bézier Mesh

Solution: Surface Current Density

Independent RWG MoM Solution

LCN Order 3, ACA (Tolerance3)

LCN Order 6, LUD (Reference)

5GHz (outer radius=36.67\(\lambda\))

Ref Solution (Order3)

Error of J from Order1 to Order2, Ref Solution=Order3
Study of Accuracy:
Torus ACA Compression Stats

![Graph showing Compression (%) vs. # Unknowns (x 1000). The graph compares Matrix Fill and LU methods. Matrix Fill shows a higher compression rate than LU across different # Unknowns values.]
Study of Efficiency: Torus at 6.5 GHz

- 55,296 unknowns (22.8 GB)
- Order 3 (Fixed)
- Reference Solution (LUD)

- Z compression = 84% (3.6 GB)
- LU compression = 74% (5.9 GB)
- ACA (Tolerance = 1e-5)
Study of Efficiency: Torus at 5 GHz

- 331,776 unknowns (820 GB)
- Order 5 (Fixed)
- Z compression = 93.7% (52 GB)
- LU compression = 89.6% (85 GB)
- ACA (Tolerance = 1e-5)

$36.67\lambda$
Study of Efficiency: B2-Aircraft at 1.25 GHz

- 55,200 unknowns (23 GB)
- Orders 2,3 (Mixed)
- Reference Solution (GMRES)

- Z compression = 81% (4.3 GB)
- LU compression = 75% (5.7 GB)
- ACA (Tolerance = 1e-5)
- 1 Tesla K20m GPU: 25 Minutes
- Intel(R) Xeon(R) CPU E5-2640 0 @ 2.50GHz (using 24 threads): 38 Minutes
- CPU vs. GPU Speedup = ~1.5 Times
Study of Efficiency: B2-Aircraft at 2 GHz

• ~120k unknowns @ order 9 (107 GB)
• Z compression = 83.7% (17.5 GB)
• LU compression = 70.6% (31.5 GB)
• ACA (Tolerance = 1e-6)
• Dual 8-core Intel(R) Xeon(R) CPU (using 24 threads):
  • ~21 hours (No GPUs)
Conclusions

• Error-controllable solution via ACA tolerance with high-order convergence

• LCN compresses very well in ACA scheme due to point-based expansion of currents

• Provides two mechanisms for solving large geometries (fast solver and higher orders)

• Double precision implementation should lead to further improvements in convergence